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INERTIAL IMPACTION EFFICIENCY OF CYLINDRICAL COLLECTORS BY DIGITAL TECHNIQUES AND EFFECTS OF PARTICLE SIZE DISTRIBUTIONS

Arthur K. Stuempfle, et al

Edgewood Arsenal
Aberdeen Proving Ground, Maryland

October 1974

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by

Arthur K. Stuempfle
Claude R. Pellegrino

Chemical Laboratory

October 1974

DEPARTMENT OF THE ARMY
Headquarters, Edgewood Arsenal
Aberdeen Proving Ground, Maryland 21010

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The theory of inertial impaction of particles on cylinders has been analyzed and used to develop standard inertial impaction efficiency curves. Unique exponential functions have been generated by a digital computer that accurately fit the inertial impaction theory with a maximum relative error of less than 1%. Interpolative routines have been adapted for the computer program to obtain inertial impaction efficiency predictions for all inertial parameter and velocity field scaling parameter values in the range of $0.13 < K \leq 300$ and $0 \leq \phi \leq 10{,}000$. The complete computer program with examples and solutions of test cases are presented in the appendix.

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20. ABSTRACT

The Weibull distribution function has been adapted to analyze the effects of particle size distributions on the impaction efficiency of cylinders. Use of the mass median diameter to characterize a particle size distribution is unsatisfactory for predicting the impaction efficiency from heterogeneous aerosols. Use of the particle size distribution to predict efficiency of impaction yields good agreement between theory and experiment.

PREFACE

The work described in this report was authorized under Task 1W162116A08402, Chemical Test and Assessment Technology. This work was started in February and completed in May 1974.

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INERTIAL IMPACTION EFFICIENCY OF CYLINDRICAL COLLECTORS BY DIGITAL TECHNIQUES AND EFFECTS OF PARTICLE SIZE DISTRIBUTIONS

I. INTRODUCTION.

The deposition of particles of intermediate size (10 to 200 μm diameter) on collectors in a flow stream is principally based on the inertial impaction mechanism. Inertial impaction of particles on collectors is of interest in diversified areas such as crop dusting, mosquito spraying, air pollution control, aircraft icing, particulate buildup on heat exchangers, and in practically any circumstance where matter in particulate form is removed from a transport fluid. The efficiency with which the particles are removed from the flow stream is a function of the particle size, collector size, and flow field conditions. For a stationary cylinder in a moving airstream, the impaction efficiency is defined as the ratio of the cross-sectional area of the upstream envelope containing the trajectories of the particles which intersect the collector surface to the cross-sectional area of the cylindrical collector normal to the direction of flow. Estimates of the impaction efficiency for a given set of conditions can be found by computing the trajectories of the particles that challenge the collector or by graphically determining the efficiency from curves constructed from the point-by-point trajectory calculations. These methods are adequate when only a few data points are of interest but the technique becomes tedious when estimating efficiencies for a variety of possible impaction conditions. In addition, the aerosol that challenges the collector generally consists of a wide range of sizes depending on the method used for particle generation. It is recognized that, if a single parameter, such as the mass median diameter (MMD), is used to characterize the particle size distribution for computational purposes, the expected theoretical impaction efficiency can be grossly different from the experimental efficiency. These differences can be attributed in part to the turbulent nature of the flow field and to the nonlinear character of the inertial impaction efficiency curves. The objectives of this study have been to develop a computer program that rapidly calculates the inertial impaction efficiency of cylindrical collectors for any given set of laminar flow conditions and to devise a technique to assess the effects of a particular particle size distribution on impaction efficiency.

II. BACKGROUND.

The theory of inertial impaction of spherical particles on cylindrical collectors in an ideal flow field is based on a numerical solution of the equations of motion of the particles undergoing transport around the bluff body. Development of the theory and associated digital computer techniques has been described in a previous study[1] and provides the basis for this report. Langmuir and Blodgett[2] proposed the following dimensionless form of the equations of motion for a particle in Cartesian coordinates.

$$\frac{dv_x}{d\tau} = \frac{C_D Re}{24} \frac{1}{K} (u_x - v_x) \tag{1}$$

$$\frac{dv_y}{d\tau} = \frac{C_D Re}{24} \frac{1}{K} (u_y - v_y) \tag{2}$$

where

v_x, v_y = particle velocity components normalized by the free-stream velocity

u_x, u_y = airstream velocity components normalized by the free-stream velocity

$\overline{U}$ = free-stream velocity at an infinite distance from the cylinder surface

C_D = drag coefficient for spherical particles in fluid

$Re = \frac{\rho_a d_p \bar{v}}{\mu}$ = particle Reynolds number with respect to local relative velocity

ρ_a = fluid density

μ = fluid viscosity

d_p = particle diameter

$K = \frac{\rho d_p^2 \bar{U}}{18 \mu R}$ = inertial parameter of particle (3)

ρ = particle density

R = cylinder radius

$\tau = \frac{\bar{U}}{R} t$ = time scale

The inertial parameter, K, is a measure of the inertia of the particle and relates to the magnitude of the external force required to cause a change in its direction of motion. If Stokes' law of resistance is assumed, the inertial parameter represents the ratio of the "stopping" distance of a particle projected with velocity $\bar{U}$ into still air to the radius of the cylinder. A second dimensionless parameter proposed by Langmuir and Blodgett is the velocity field scaling parameter, ϕ, originally defined in terms of the free-stream particle Reynolds number. The ϕ parameter is used to calculate the magnitude of the instantaneous Reynolds number of the particle at any point in the flow field and can be related to the Reynolds number of the cylindrical collector as follows:

$$\phi = \frac{Re_o^2}{K} \frac{18 \rho_a^2 \bar{U} R}{\mu \rho} = \frac{9 \rho_a}{\rho} (Re_c) \tag{4}$$

where

Re_o = $\frac{d_p \rho_a \bar{U}}{\mu}$ = free-stream Reynolds number of the particle

Re_c = free-stream Reynolds number of the cylindrical collector

The magnitude of the ϕ parameter at any point in the particle path is a measure of the deviation from Stokes' law due to the forces acting on the particle.

The starting point for a mathematical description of the fluid flow field around the collector is the Navier-Stokes equation. A solution to the general Navier-Stokes equation is possible after a number of limiting assumptions are made with respect to the fluid and flow conditions. One form of solution can be found if the fluid is assumed to be ideal; i.e., constant density, irrotational, and without viscosity. The flow pattern around the cylinder under steady-state conditions is dependent on the Reynolds number of the collector. Where the Reynolds number of the cylinder is one thousand or greater ($\phi \geq 10$ in air), the flow can be considered ideal in the absence of turbulence and the flow field is adequately described by potential theory for an incompressible fluid. The streamlines for flow outside the cylinder can be obtained and the airstream velocity components can be written in dimensionless terms as simple functions of the reduced position coordinates[1]; namely,

$$u_x = 1 + \frac{y^2 - x^2}{(x^2 + y^2)^2} \tag{5}$$

$$u_y = - \frac{2xy}{(x^2 + y^2)^2} \tag{6}$$

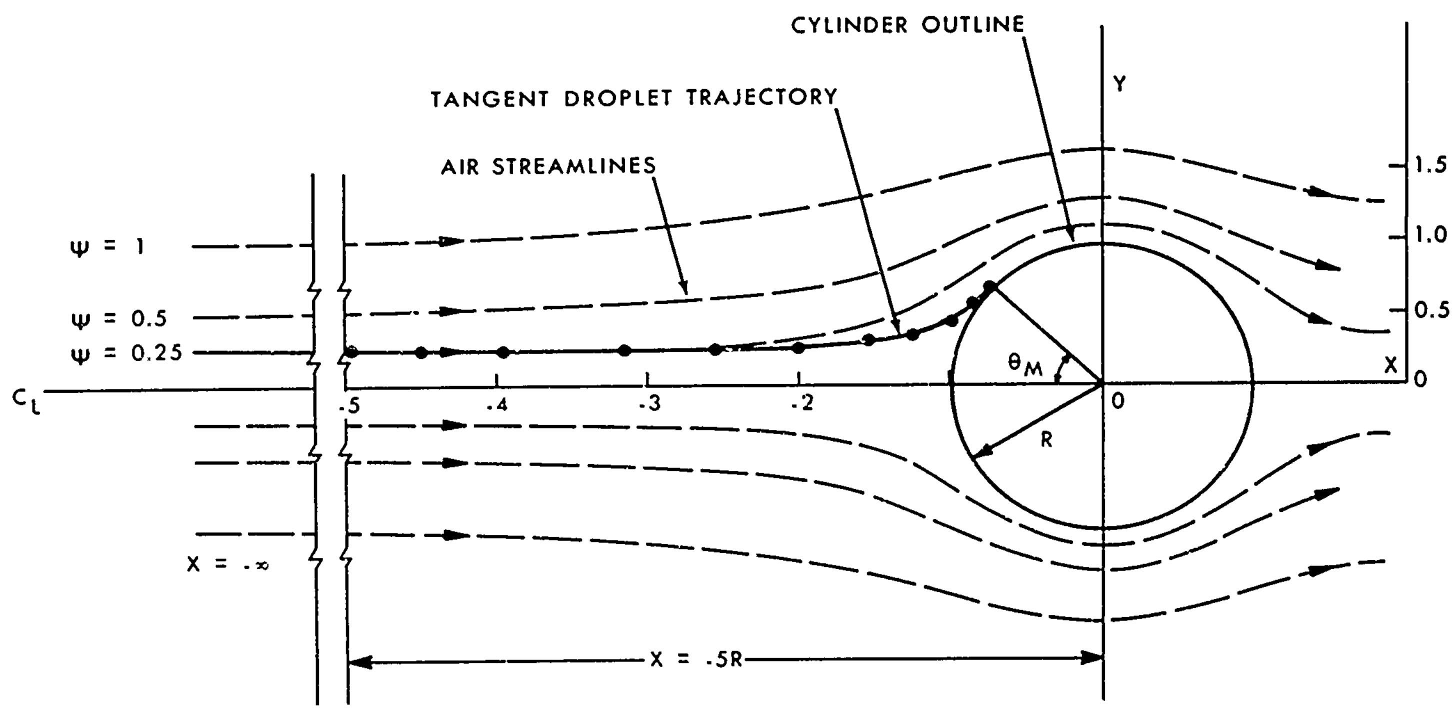

Figure 1. Coordinate System for Cylinder in Potential Flow Field

where the Cartesian coordinates have been normalized with respect to the cylinder radius.

A numerical solution of the differential equations of motion given above requires the instantaneous drag force on the particle to be estimated as the trajectory is developed. The steady-state drag coefficient of the sphere is a function of the Reynolds number, and the reliable experimental data as tabulated by Fuchs[3] have been interpolated to obtain these estimations. The probable error in the drag coefficient for solid spheres is claimed by Fuchs to be less than 4% over the Reynolds number range of 0.01 to 500. These data are nearly identical to the experimental data cited by Schlichting[4] as adopted and reported by Hussein and Tabakoff.[5] Over the Reynolds number range of 0.10 to 7.0, the earlier approximations used by Langmuir and Blodgett differ from the current drag coefficient data.

The Cartesian coordinate system used in calculating the trajectory of particles around the cylindrical collector is identical to that of Brun, Lewis, Perkins, and Serafini[6] and is shown in figure 1. The motion of the particles is in a plane perpendicular to the cylinder axis which is the origin of the coordinate system. Theoretically, the maximum angle of impingment, ϑ_M, is the angle beyond which no deposition occurs by the inertial mechanism.

Several simplifying assumptions have been made in the derivations and in computing the trajectory of a particle. These assumptions include the following:

1. At an infinite distance upstream of the cylinder, the particles have horizontal and vertical velocity components equal to the free-stream air.

2. The particles are spherical, monometric (uniform size), and monodisperse (single particles), and they do not evaporate or deform.

3. Gravitational, electrostatic, and any other external forces are negligible.

4. The particle radius is negligible with respect to the cylinder radius. (For interception effect considerations, see previous publication.[7])

5. The boundary layer about the cylinder surface does not affect the particle trajectory.

6. The airflow around the cylinder is described as ideal and without circulation and is unaffected by the presence of the particles.

7. The instantaneous drag force coefficient for the particle is given by the steady-state data and is not subjected to acceleration effects.

8. All particles that strike the collector adhere to it.

Results of the digital computer trajectory calculations to determine the efficiency of impaction of spherical particles on cylindrical collectors in an ideal flow field as a function of the K and ϕ scaling parameters are shown in figure 2. The impaction data for inertial parameter values less than one ($K \leq 1$) are displayed in figure 3. In addition to the impaction efficiency, the maximum-angle-of-impingement data are plotted in figure 4. The theory has been experimentally verified under laminar flow conditions for a wide range of inertial parameter and velocity field scaling parameter values.[8-11] In general, however, the theory is inappropriate for accurately predicting the impaction efficiency of particles possessing small inertial parameter values on collectors in fluid flows that exhibit turbulence intensity levels above 7.5% or when the Reynolds number of the collector is less than approximately one thousand ($\phi \simeq 10$ in air).[11] Further, the collection efficiency of a cylinder that is challenged by a distribution of particle sizes is not accurately predicted by a single particle parameter such as the mass median diameter.[9,12] For many practical circumstances, though, the theory is quite sufficient, and this study has been undertaken so that impaction efficiency predictions can be rapidly made by computer with minimum input data requirements or computer programming background.

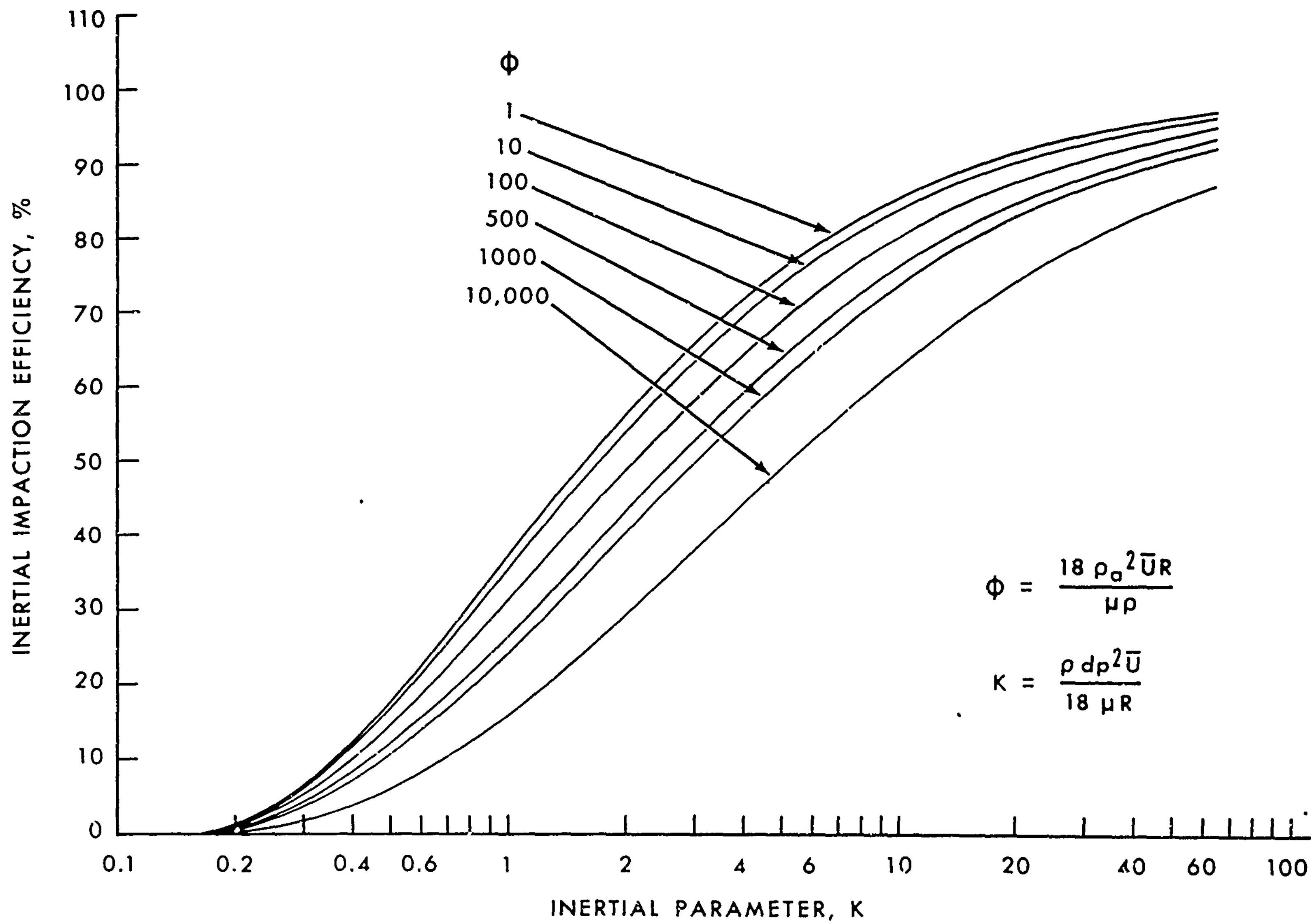

Figure 2. Inertial Impaction Theory

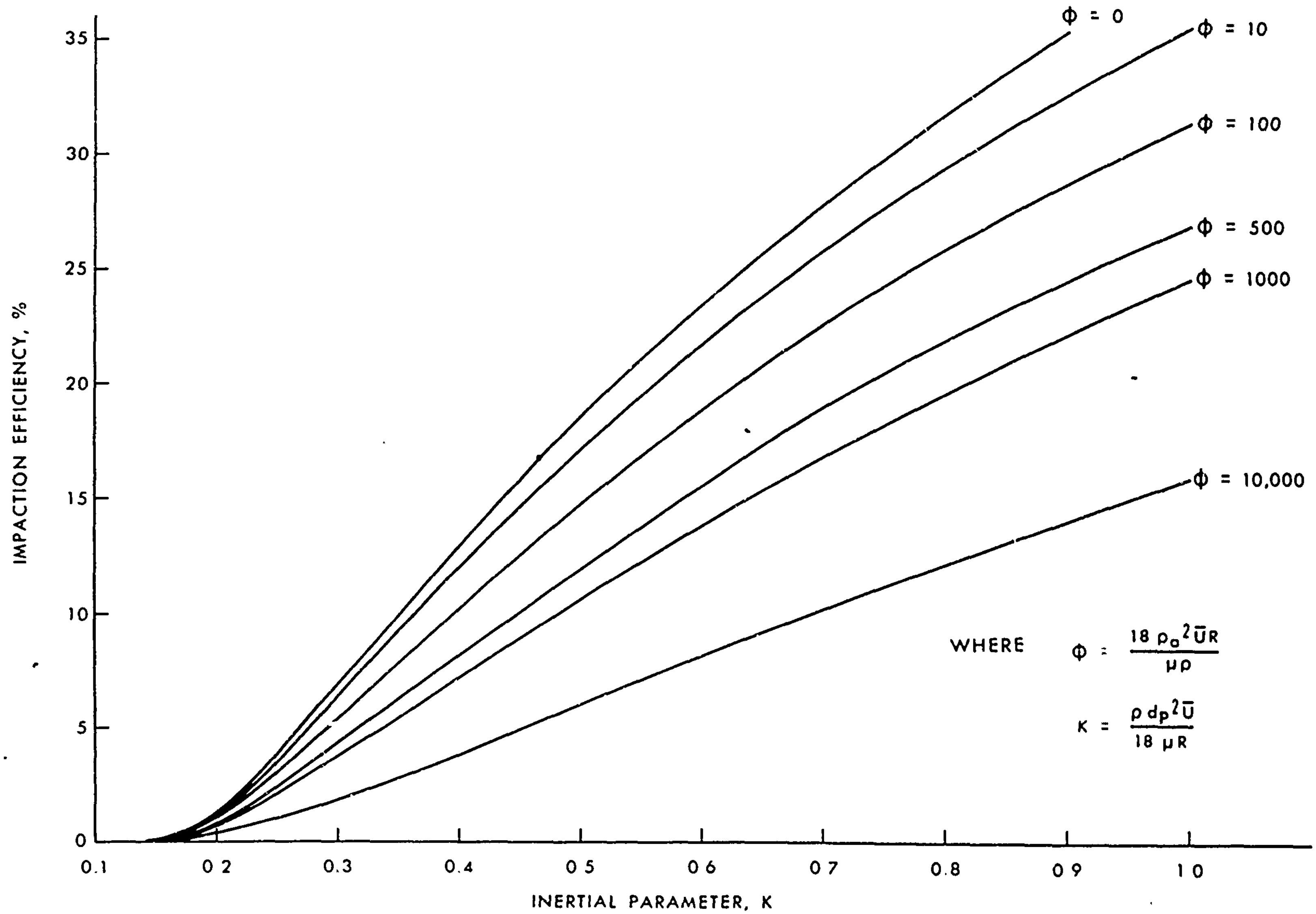

Figure 3. Impaction Efficiency for Cylinder for K ≤ 1.0

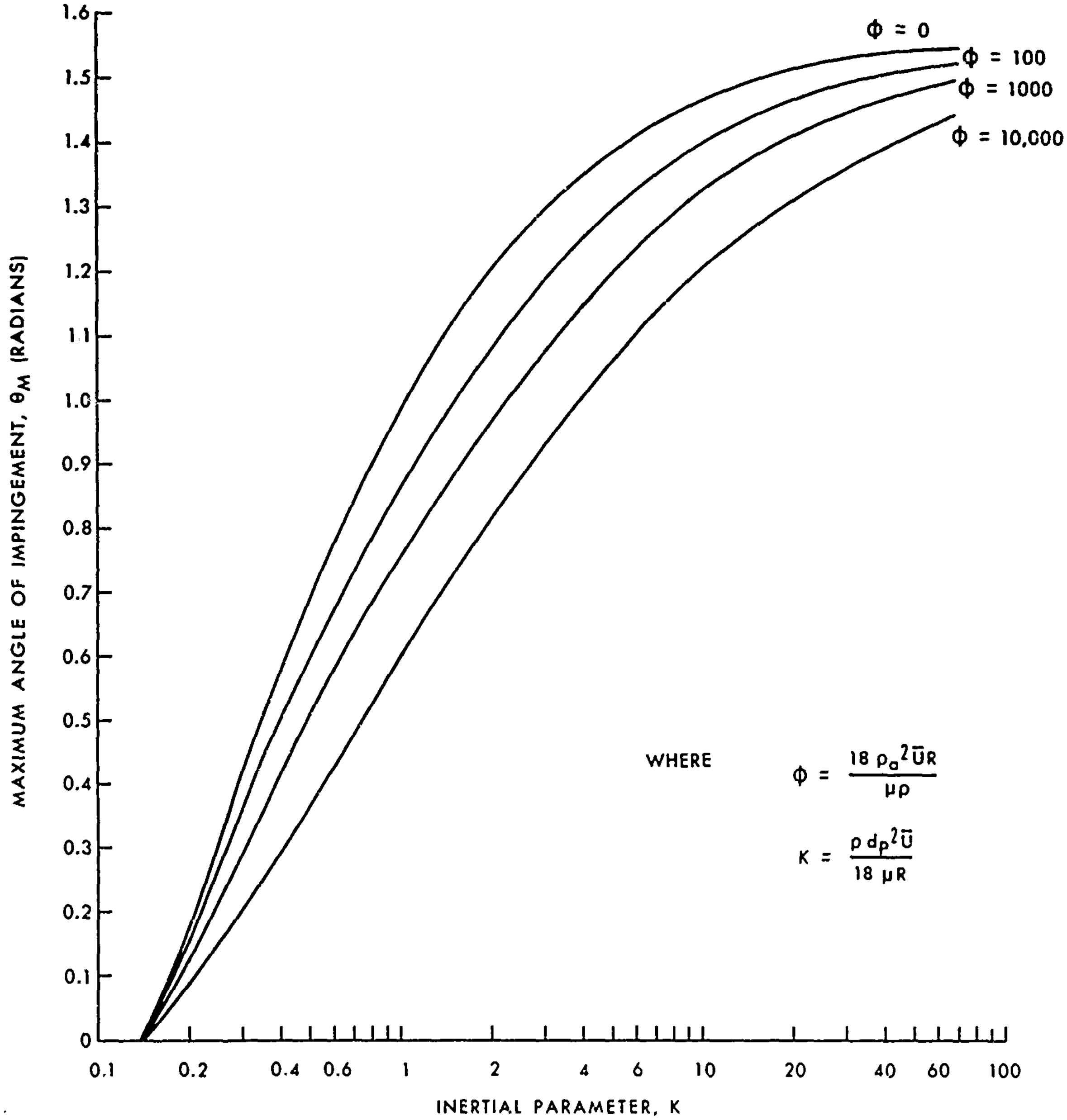

Figure 4. Maximum Angle of Impingement on Cylinders

III. METHODS AND RESULTS.

Results of the point-by-point trajectory calculations define the inertial impaction theory as shown in figures 2 and 3 and provide the impaction efficiency as a function of the K and ϕ scaling parameters. These data serve as the standards for which unique exponential functions have been obtained to fit the curves. The analytical expressions for the standard ϕ curves can then be interpolated to determine the impaction efficiency for any nonstandard K and ϕ value. A digital computer program has been developed to perform these calculations and only a minimum of input data is necessary for operation.

The standard velocity field scaling parameter curves selected for fit include ϕ = 0; 1; 2.2; 5; 10; 22; 50; 100; 200; 500; 1,000; 2,210; 5,000; and 10,000. Each curve has been subdivided into four inertial parameter ranges and each portion, respectively, has been fitted by the same form of the unique exponential functions. Any differences that occur between the computed efficiency along a standard ϕ curve and the point-by-point impaction efficiency data are in the third or higher significant digit position and the maximum relative error observed has been less than 1%.

Lagrange's interpolation formula has been used to compute impaction efficiency values for velocity field scaling parameters enclosed within the aforementioned standard ϕ values that are equal to or greater than one. The formulation used in this study has been defined as

$$E_\phi = \sum_{i=1}^{M} E_{\phi_i} \frac{\prod\limits_{\substack{j=1 \\ i \neq j}}^{M} (\ln\phi - \ln\phi_j)}{\prod\limits_{\substack{j=1 \\ i \neq j}}^{M} (\ln\phi_i - \ln\phi_j)} \tag{7}$$

where

$$E_{\phi_i} = 1.0/\left\{\exp\left[\exp(A_N(\ln K)^N + A_{N-1}(\ln K)^{N-1} + \ldots + A_2(\ln K)^2 + A_1 \ln K + A_0)\right]\right\} \tag{8}$$

and

$0.13 < K \leq 64$

$\phi_i, \phi_j = 1; 2.2; 5; 10; \ldots 5{,}000; 10{,}000$

$i, j = 1, 2, \ldots, 13$

$M = 1, 2, \ldots, 13$

$N = 4\text{–}7$

If $64 < K \leq 300$,

$$\phi_i, \phi_j = 1; 5; 22; 100; 500; 2{,}210; 10{,}000$$

$$i, j = 1, 2, ..., 7$$

$$M = 7$$

where K = inertial parameter valut of interest; $0.13 < K \leq 300$.

One advantage of Lagrange's interpolation formula over other interpolative routines is that equidistance values of the independent variable are not necessary to obtain a reliable value of the dependent variable. For ϕ values between 0 to 1, where the potential flow assumption may not be valid, a linear interpolation has been employed.

Comparisons have been made between impaction efficiency values obtained by the interpolative method, and the point-by-point trajectory technique and relative errors are less than 1% when the absolute impaction efficiency is greater than 5%. For impaction efficiencies less than 5%, the largest relative error observed has been within 10% of the point-by-point trajectory calculated efficiency value and is due principally to round-off (four-place accuracy). The trajectory calculations had been terminated at the inertial parameter value of 64. However, the interpolative routine extends this range to K values of 300 where the impaction efficiency curves asymptotically approach the theoretical limiting efficiency of 100% for point-mass particles. Consequently, for all practical purposes, the interpolative routine provides an accurate prediction of the impaction efficiency of spherical particles on cylindrical collectors over an extensive range of inertial parameter values. A complete and detailed description of the digital computer program developed in this study is contained in the appendix.

Experimental verification of the inertial impaction theory has been accomplished over a wide range of parameter values. The theory turns out to be remarkably accurate for the monometric (single-size) particles used in the experimental efforts especially in view of the assumptions that have been made regarding fluid properties and flow conditions.

In practical applications, however, the aerosol that impacts on a collector will seldom consist of particles of uniform size. The size distribution of the aerosol is a function of the methods and techniques used in generation of the particles. A single empirical or theoretical equation does not exist that can universally predict the particle size distribution resulting from the dispersion of liquids by spray nozzles, hot and cold gas atomizers, and explosive disseminators or the dispersion of solids from mechanical dispersers.

A vast literature has been developed by various authors in their attempts to obtain mathematical distribution functions that characterize experimentally generated particle size distributions. Familiar examples include the Nukiyama-Tanasawa equation for drop sizes in sprays generated by air atomization, the Rosin-Rammler equation for pulverized coal, Roller's formula for powder materials, the normal distribution function for symmetrically distributed particles of narrow size range such as plant spores, and the log-normal distribution for a large number of condensation, natural, and mechanically generated aerosols. However, the intent of this study is not to generalize on particle size distributions or delve into their specific merits and methods but rather to examine the necessity of accounting for the particle size distribution when predicting the impaction efficiency of a particular cylindrical collector.

A simple technique used to obtain a size distribution has been to microscopically observe and measure a representative sample of an aerosol population. The number of particles that lie between radius r and r + dr can be found as a function of the radius. The fraction of the total number of particles that lie in an interval is given as $dn = f(r)dr$

where $\int_0^\infty f(r)dr = 1.$

The relative frequency distribution curve is found by plotting the resulting data representing dn, and the various distribution functions mentioned above can provide a mathematical expression of the function, f(r).

Distribution functions can be mathematically expressed and used in a number of ways depending on the problem requirements. In some instances, the cumulative fraction of particles having radii greater or less than some radius is convenient. Thus, respectively,

$$F(r) = \int_r^\infty f(r)dr \text{ or } F(r) = \int_0^r f(r)dr$$

In addition to the number distribution, the mass distribution function can be determined and is useful in many practical applications. The mass fraction of particles having radii between x and x + dx is written as $df = f(x)dx$ and $\int_0^\infty f(x)dx = 1.$ The mass distribution function is related to the number distribution function as

$$f(x) = \omega m_r f(r)$$

where

m_r = mass of particle with radius r

ω = proportionality constant

With respect to impaction of particles on collectors, one is usually interested in the mass distribution function because the dose acquired by the collector is dependent on the aerosol mass that is deposited over the sampling period. In order to estimate the mass collected, an average particle size is generally computed and applied to the inertial impaction theory. The objectives of using an average diameter or other measure of central tendency are to provide a single number that will simply describe the behavior of the entire aerosol population and to eliminate an extraordinary number of tedious calculations. The problem is that the inertial parameter value computation makes use of the square of the particle diameter and, further, the impaction curves are a nonlinear function of the inertial parameter. Consequently, an accurate estimation of impaction efficiency or mass deposit can only be expected when a single particle size estimator is used if the particle size distribution is over a very narrow range.

In order to demonstrate the effect that a particle size distribution has on the impaction efficiency, experimental data are required for comparison purposes. Most of the experimental work that has been performed in the past to verify the inertial impaction theory has been conducted with monometric (single-size) aerosols. One notable exception is that of Landahl and Herrmann.[12] Several authors refer to these data but they consider the results to be of limited value due to the use of heterogeneous aerosols, especially at small values of the inertial parameter.

Landahl and Herrmann studied the deposition of aerosols on wires and cylinders with particles generated by an impinger-type atomizer and by a "modified Brink's nozzle." The size distributions were obtained by cascade impactors, and microscopic observations and representative distributions of the clouds produced are illustrated in the following table:

Percent*	Diameter: Small particle cloud	Diameter: Medium particle cloud	Diameter: Large particle cloud
	μm	μm	μm
5	(0.8)	1.8	5
10	1.0	2.9	8
50	4.0	13	28
90	12	40	58
98	(20)	55	75

*Percent of mass less than stated diameter.

These data are not easily fitted or well represented by the aforementioned distribution functions. However, a general statistical distribution function that has found wide applicability recently is the Weibull distribution function.[13-15] This distribution function has been successfully used in processes involving limits and maxima/minima problems that include lifetime and failure rate distributions of electrical systems. It has also been used for describing bounded particle size distributions.

The cumulative distribution function of the Weibull distribution has been applied to the Landahl and Herrmann data and has been expressed in the following form:

$$F(x) = 1 - \exp\left[-\left(\frac{x-\gamma}{\eta-\gamma}\right)^{\beta}\right] \tag{9}$$

where

$F(x)$	=	cumulative mass fraction
x	=	particle diameter (μm)
γ	=	minimum size; location parameter (μm)
η	=	characteristic size; scale parameter (μm)
β	=	shape parameter

The trial-and-error method of estimating the Weibull parameters has been described by Nelson[15] and a digital computer program has been developed[16] to estimate the parameters of the distribution function for the small, medium, and large particle clouds. The parameter approximations obtained by the principle of maximum likelihood method turn out to be as follows:

Parameter	Small cloud	Medium cloud	Large cloud
η	5.3053	17.9678	34.4356
β	0.9532	1.1183	1.7461
γ	0.5919	0.5269	-1.7375

Evaluating the cumulative mass function for the experimental particle diameters to assess the Weibull distribution fit yields the following comparisons:

Experimental F(x)	Small cloud		Medium cloud		Large cloud	
	x	F(x) calculated	x	F(x) calculated	x	F(x) calculated
%	μm	%	μm	%	μm	%
5	0.8	5	1.8	5	5	5
10	1.0	9	2.9	10	8	10
50	4.0	52	13	50	28	51
90	12	90	40	92	58	91
98	20	98	55	97	75	98

As observed, an adequate fit is obtained for all clouds considering the limited available data and the wide size distributions that are involved. It must be emphasized again that the objective is not to find the "correct" distribution function to describe a particular particle size distribution but rather to approximate the function as simply as possible and to test whether the resultant distribution significantly affects the impaction efficiency predictions.

Given the above distribution functions and approximate values for the Weibull parameters, one can solve equation 9 for the particle size as a function of the cumulative mass fraction, F(x); namely,

$$x = \gamma + (\eta - \gamma) \left[\ln \left(\frac{1}{1 - F(x)} \right) \right]^{1/\beta} \tag{10}$$

Subsequently, by selecting appropriate values for F(x), the average particle size associated with a given particle mass interval can be determined by difference. For example, if 30% of the cloud mass has particle sizes below 7.4 μm and 31% of the cloud mass has particles less than 7.6 μm in diameter, then 1% of the cloud mass is represented by particles with an average diameter of approximately 7.5 μm. The impaction efficiency for each size interval can then be computed and an average efficiency can be obtained for the entire distribution.

A computer program has been written to perform the tedious computations for each 1% mass interval, and the average distribution impaction efficiency is found based on 96% of the cloud mass. In addition, the program provides the impaction efficiency based strictly on the mass median diameter of the cloud. The only supplementary imput requirements necessary for use of the distribution function option to the computer program are the Weibull parameters η, β, γ.

The sample distribution functions have been used to compute average impaction efficiencies for comparison with the experimentally obtained efficiencies of Landahl and Herrmann. The results of these computations are included in table 1. The velocity field scaling parameter values for the Landahl and Herrmann tests are very low and theoretically the potential flow assumptions regarding the fluid flow field should not apply. Further, Landahl and Herrmann state that the airflow was turbulent, but the degree of turbulence is not indicated. In addition, the illustrative particle size distributions provided by Landahl and Herrmann do not correspond with their reported efficiency data except for the 13-μm-MMD case. Nevertheless, examination of table 1 clearly shows that the theoretical impaction efficiencies for the distribution functions more closely predict the observed impaction efficiencies than the mass median diameter computed efficiencies for almost all circumstances. This is especially true for the 13-μm-MMD case where the assumed particle size distribution apparently matches the experimental mass median diameter and the theoretical efficiencies turn out to be considerably more accurate than the mass median diameter prediction. Note that the small- and large-cloud particle size distributions have mass median diameters that exceed the experimental mass median diameters and, therefore, tend to overestimate the theoretical efficiencies from the distribution function. Thus, to summarize the results, when the particle size distribution is used to compute the average impaction efficiency for the cloud, the prediction corresponds much more closely to the observed efficiency than that of a simple mass median diameter prediction. The predictions are relatively accurate when the particle size distribution is known as evidenced by the 13-μm-MMD test case.

IV. DISCUSSION.

The important conclusion that can be drawn from the theoretical and experimental data presented in table 1 is that use of the mass median diameter to describe a particle size distribution will seldom result in an accurate prediction of the impaction efficiency for the cloud. In general, at large values of the inertial parameter, K, the mass median diameter overestimates the impaction efficiency of the distribution, whereas, for small values of the parameter where the theoretical efficiency for the mass median diameter is less than 1%, the efficiency can be underestimated by orders of magnitude. These differences are due to the nonlinear character of the inertial impaction theory. However, when the particle size distribution is considered in the predictions, the agreement between theory and experiment is much better.

Calculations were performed using the typical particle size distributions previously given to determine the range of inertial parameter values over which the mass median diameter predictions were within $\simeq$10% of the average impaction efficiencies for the distributions. In general, if the mass median diameter yields a K value between 1 and 2, for all ϕ values, the effect of the distribution on efficiency need not be considered. When the inertial parameter value based on the mass median diameter lies outside this range, the average distribution efficiency will differ from the mass median diameter predicted efficiency by more than 10%. To accurately predict the impaction efficiency for heterogeneous aerosols when the K value for the mass median diameter is less than one, the particle size distribution must be taken into account. Unfortunately, data are not available to determine the degree of monometricity of the distribution that is required in order that the mass median diameter or other average particle diameter will adequately represent the population over the entire inertial impaction region of interest.

Table 1. Experimental and Theoretical Impaction Efficiencies

D Cylinder diameter	$\overline{U}$ (cm/sec) / Parameter	0.9 mph = 40.2	3 mph = 134.1			8 mph = 357.6		
		MMD (μm)	MMD (μm)			MMD (μm)		
		15	3.7	13	27	3.7	13	27
cm								
0.008	ϕ	0.022		0.074			0.198	
	EXP E (%)[a]	75	34	89	93	46	87	100
	DIST E (%)[b]	66	47	81	94	63	89	97
	MMD E (%)[c]	79	51	92	98	74	97	99
0.025	ϕ	0.069		0.232			0.619	
	EXP E (%)	50	20	69	85	31	73	84
	DIST E (%)	49	28	67	87	44	79	93
	MMD E (%)	54	18	80	94	46	91	98
0.10	ϕ	0.278		0.928			2.47	
	EXP E (%)	25	8.4	40	54	16	53	64
	DIST E (%)	25	10	45	71	21	60	82
	MMD E (%)	15	0	48	81	8.3	71	91
0.90	ϕ	2.50		8.35			22.3	
	EXP E (%)	4.3	0.70	7.6	15	2.1	19	28
	DIST E (%)	2.5	0.23	11	29	2.0	23	47
	MMD E (%)	0	0	0.21	27	0	13	52

[a]Landahl and Herrmann experimental efficiency.

[b]Average efficiency from typical particle size distributions.

[c]Efficiency from mass median diameter of typical distribution, 4, 13, and 28 μm, respectively.

It is also interesting that the predicted efficiencies are close to the experimentally measured efficiencies even though the velocity scaling parameter values are much less than 10 in some cases. In general, the lower ϕ values would lead to reduced impaction efficiencies because the flow field would tend to exhibit viscous flow properties. On the other hand, no account of interception effects has been made for the interpolation routine, and interception could occur since the particle to collector diameter ratios are substantial The mass median diameter to collector diameter ratio ranges from 4.1×10^{-4} to 3.4×10^{-1}. The effect of interception would be to substantially increase the values of the mass median diameter efficiency. However, with such large diameter ratios, the collection efficiency may not equal the theoretical impaction efficiency due to particle bounce-off and re-entrainment from the collector surface.

The Weibull distribution function is a simple mathematical function that adequately fits the data of Landahl and Herrmann. It may not apply to all particle size distributions of interest but as stated by Professor Weibull, "The only practical way of progressing is to choose a simple function, test it empirically, and stick to it as long as none better has been found."[13] The graphical technique for estimating the Weibull parameters as presented by Nelson[15] was found to be convenient and simple to use, and best estimates of the three parameter values can be rapidly found by digital computer.[16]

The computer program prepared for this study and presented in the appendix enables impaction efficiency predictions to be made over practically the entire region of interest of inertial impaction on cylinders. Four optional computational methods are provided for the user depending on the amount of input information available. Option 1 provides the impaction efficiency of a cylinder and assumes unit density particles and an ambient temperature of 20°C. The only input requirements are the mean windspeed, cylinder diameter, and particle diameter in centimeter-gram-second units. Option 2 makes use of all the variables required to compute the K and ϕ parameters defined by equations 3 and 4, but the program provides the necessary calculations based on these input values. Option 3 assumes that the user has computed the K and ϕ parameter values from equations 3 and 4 and desires only the predicted efficiency. Thus, only the K and ϕ input values are required. Option 4 considers the Weibull distribution function and requires all input data from option 2 except for the particle diameter for which the Weibull parameter estimates of η, β, and γ are substituted. The program computes the average impaction efficiency of the distribution, as well as the efficiency based on the mass median diameter of the distribution. Option 4 has been separated from the main program so that the reader could substitute his own distribution function, if required, with only a few minor program changes. A detailed explanation, sample input cards, test cases with sample outputs, and the complete program are provided in the appendix for the convenience of the reader.

The approximation methods developed in this study to calculate the impaction efficiency of a cylinder for any K and ϕ value have reduced the computational time for the point-by-point trajectory calculations used to develop the inertial impaction theory from 5.6 seconds to 13 milliseconds per data point on a digital computer comparable to a Univac 1108 computer without sacrificing the accuracy of the prediction

V. CONCLUSIONS.

1. Standard inertial impaction efficiency curves for cylinders developed from point-by-point trajectory calculations have been fitted by unique exponential functions.

2. Lagrange's interpolation formula has been adopted and applied to compute the impaction efficiency of a cylinder for any parameter values of $0.13 < K \leq 300$ and $0 \leq \phi \leq 10{,}000$.

3. The accuracy of the impaction efficiency computations by use of the digital computer program is withi · 1% of the point-by-point efficiency predictions.

4. The cumulative distribution function of the Weibull distribution has been incorporated into the digital computer program and has been used to fit the experimental particle size distributions of Landahl and Herrmann.

5. The average impaction efficiency for the particle size distributions and the mass median diameter impaction efficiencies have been compared to the experimental data obtained by Landahl and Herrmann for heterogeneous aerosols.

6. Use of a mass median diameter to characterize a particle size distribution does not accurately predict the experimentally obtained impaction efficiency. For large values of the inertial parameter, the mass median diameter overestimates the efficiency and, at small inertial parameter values, the mass median diameter significantly underestimates the impaction efficiency.

7. Consideration of the particle size distribution in impaction efficiency predictions results in good agreement between theory and experiment.

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APPENDIX

DIGITAL COMPUTER PROGRAM FOR INERTIAL IMPACTION EFFICIENCY COMPUTATIONS

I. INTRODUCTION.

The computer program IMPEF and its subroutines LAGRNG, WEIBUL, and POLY have been written for the Edgewood Arsenal Univac 1108, time-sharing, multiprocessor system.

IMPEF has four executable options. Three of these enable the user to input various particle and fluid flow conditions to obtain an inertial impaction efficiency value. The fourth option computes the average impaction efficiency for a particle size distribution generated from the Weibull distribution function and provides the impaction efficiency for the mass median diameter (MMD) of the distribution.

II. MATHEMATICAL BASIS OF PROGRAM IMPEF.

The first step in the development of the approximation methods has been to employ the point-by-point particle trajectory method* to obtain the inertial impaction efficiency curves for 14 selected velocity field scaling parameter values chosen as standards (ϕ = 0; 1; 2.2; 5; 10; 22; 50; 100; 200; 500; 1,000; 2,210; 5,000; 10,000).

Polynomial equations have been developed that accurately fit the standard ϕ curves. Each curve has bee. sectioned into four parts representing four intervals of the inertial parameter, K (0.13-0.22; 0.22-0.5; 0.5-1.0; 1.0-64.0). The polynomial coefficients are stored in the subroutine POLY. If the K parameter lies between 0 and 0.13, the impaction efficiency is assumed equal to zero based on results obtained from the point-by-point trajectory model computations.

In order to approximate the impaction efficiency for any nonstandard ϕ value over the range $1 \leq \phi <$ 10,000, Lagrange's interpolation formula has been applied after assuming a functional relationship exists among the efficiencies at a given K value. The advantage of Lagrange's interpolation routine is that equidistant values of the independent variable are not necessary to obtain a reliable result. A linear interpolation has been used for those cases where the ϕ value falls between 0 and 1.

Lagrange's routine is utilized with alternating standard ϕ curves (1; 5; 22; 100; 500; 2,210; 10,000) to compute efficiencies for inertial parameter values in the extrapolated range of $64 < K \leq 300$.

The adaptation of Lagrange's interpolation formula used in the LAGRNG subroutine of the program IMPEF is written as:

$$E_\phi = \sum_{i=1}^{M} E_{\phi_i} \frac{\prod\limits_{\substack{j=1 \\ i \neq j}}^{M} (\ln \phi - \ln \phi_j)}{\prod\limits_{\substack{j=1 \\ i \neq j}}^{M} (\ln \phi_i - \ln \phi_j)}$$

*Stuempfle, A. K. EATR 4705. Impaction Efficiency of Cylindrical Collectors in Laminar and Turbulent Fluid Flow. Part I. Inertial Impaction Theory. March 1973. UNCLASSIFIED Report.

where

$$E_{\phi_i} = 1.0/\{\exp[\exp(A_N(\ln K)^N + A_{N-1}(\ln K)^{N-1} + \ldots + A_2(\ln K)^2 + A_1 \ln K + A_o)]\}$$

and K = inertial parameter value of interest; $0.13 < K \leq 300$.

If $0.13 < K \leq 64$,

ϕ_i, ϕ_j = 1; 2.2; 5; 10; 22; 50; 100; 200; 500; 1,000; 2,210; 5,000; 10,000

i, j = 1, 2, ..., 13

M = 1, 2, ..., 13

N = 4, 5, 6 or 7

If $64 < K \leq 300$,

ϕ_i, ϕ_j = 1; 5; 22; 100; 500; 2,210; 10,000

i, j = 1, 2, ..., 7

M = 7

A_M's are the coefficients obtained from the fit of the standard ϕ curves and are given in the subroutine POLY.

III. EXPLANATION OF INPUT FOR PROGRAM IMPEF WITH EXAMPLES.

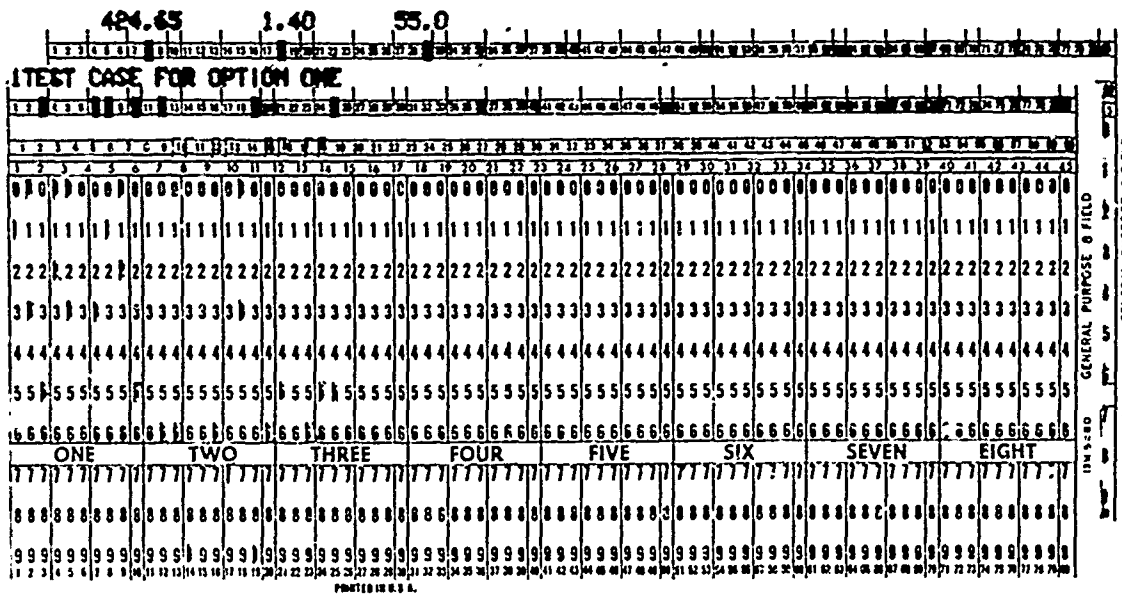
Option One.

Computes the velocity field scaling parameter, φ, the inertial parameter, K, and the inertial impaction efficiency assuming an ambient temperature of 20°C and unit density particles. Requires input of particle diameter, cylinder diameter, and free-stream wind velocity.

Format for Input Cards

Card	Column	Format	Symbol	Denotation	Unit
1	1	I1	IOP	Option	
	2-80	79A1	ID	Data identification	
2	1-10	F10.0	U	Free-stream velocity	cm/sec
	11-20	F10.0	D	Collector diameter	cm
	21-30	F10.0	PDM	Particle diameter	microns

Sample Input



Sample Output

```
TEST CASE FOR OPTION ONE
PARTICLE DENSITY = 1.0000+00 GM/CC      FLUID DENSITY = 1.2047-03 GM/CC
FLUID VISCOSITY = 1.8100-04 POISE      COLLECTOR DIAMETER = 1.4000 CM
FREE-STREAM VELOCITY =  424.650 CM/SEC
PARTICLE DIAMETER =  55.000 MICRONS
VELOCITY FIELD SCALING PARAMETER, PHI =     42.9023
INERTIAL PARAMETER, K =     5.6326
INERTIAL IMPACTION EFFICIENCY = .7353164
```

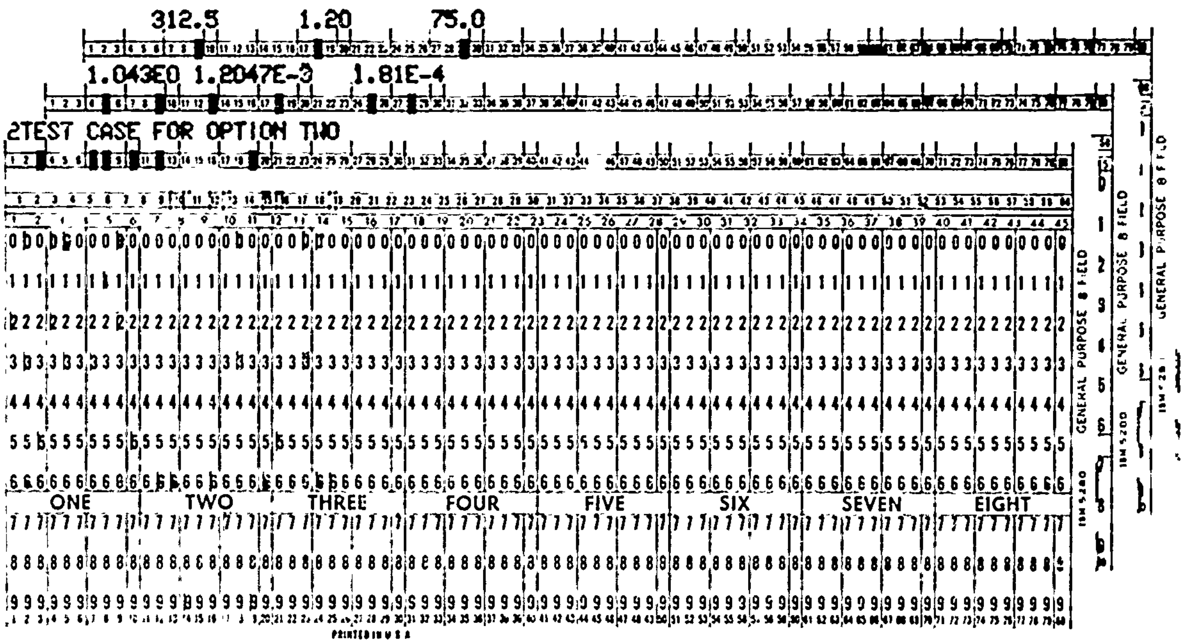
Option Two.

Computes the velocity field scaling parameter, ϕ, the inertial parameter, K, and the inertial impaction efficiency for the applicable input conditions. Requires input of particle diameter and density, collector diameter, fluid density and viscosity, and free-stream wind velocity at the applicable temperature.

Format for Input Cards

Card	Column	Format	Symbol	Denotation	Unit
1	1	I1	IOP	Option	
	2-80	79A1	ID	Data identification	
2	1-10	E10.0	RHO	Particle density	gm/cc
	11-20	E10.0	RHOA	Fluid density	gm/cc
	21-30	E10.0	RMU	Fluid viscosity	poise
3	1-10	F10.0	U	Free-stream velocity	cm/sec
	11-20	F10.0	D	Collector diameter	cm
	21-30	F10.0	PDM	Particle diameter	microns

Sample Input



Sample Output

```
TEST CASE FOR OPTION TWO
PARTICLE DENSITY = 1.0430+00 GM/CC     FLUID DENSITY = 1.2047-03 GM/CC
FLUID VISCOSITY = 1.8100-04 POISE     COLLECTOR DIAMETER = 1.2000 CM
FREE-STREAM VELOCITY =  312.500 CM/SEC
PARTICLE DIAMETER =  75.000 MICRONS
VELOCITY FIELD SCALING PARAMETER, PHI =     25.9453
INERTIAL PARAMETER, K =    9.3793
INERTIAL IMPACTION EFFICIENCY = .8221419
```

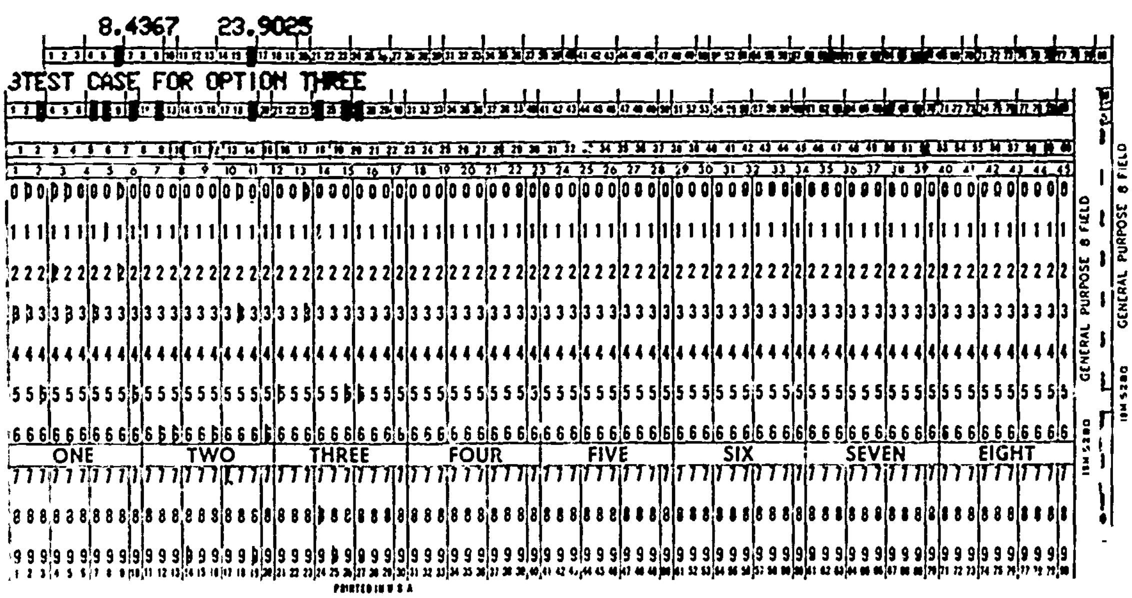
Option Three.

Computes the inertial impaction efficiency for a given set of K and ϕ input parameter values.

Format for Input Cards

Card	Column	Format	Symbol	Denotation
1	1	I1	IOP	Option
	2-80	79A1	ID	Data identification
2	1-10	F10.0	PHI	Velocity scaling parameter
	11-20	F10.0	RK	Inertial parameter

Sample Input



Sample Output

```
TEST CASE FOR OPTION 3
VELOCITY FIELD SCALING PARAMETER, PHI =      8.4367
INERTIAL PARAMETER, K =  23.9025
INERTIAL IMPACTION EFFICIENCY = .9223695
POTENTIAL FLUID FLOW MAY NOT APPLY
```

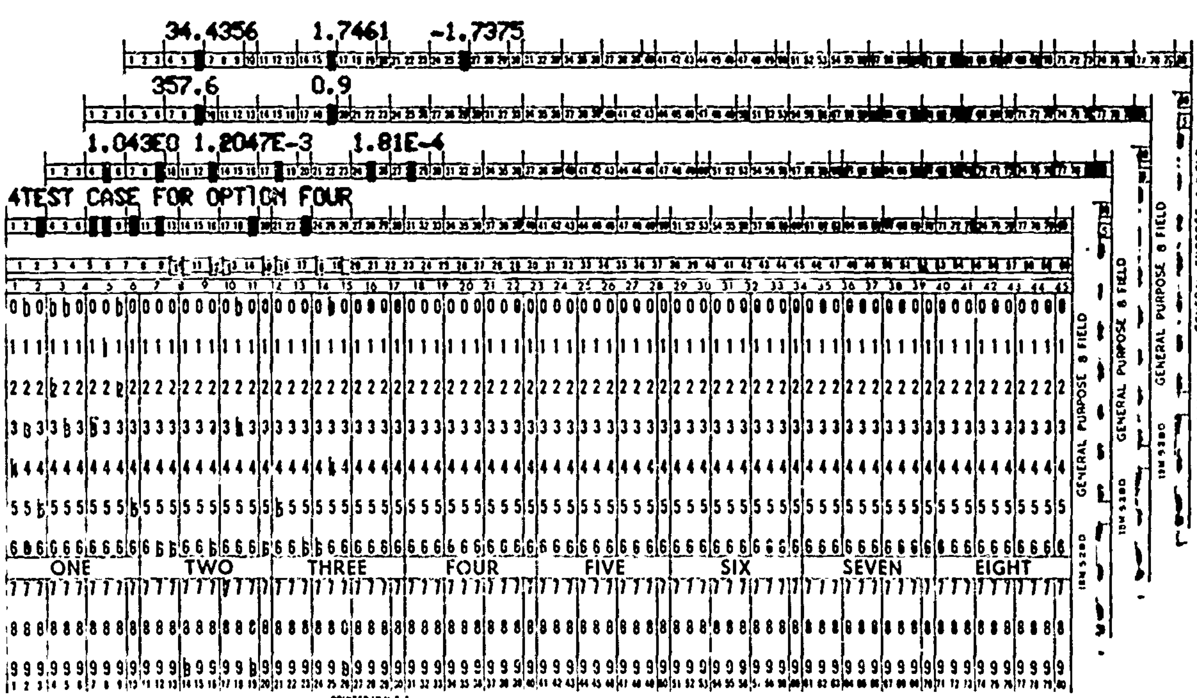
Option Four.

Computes a particle size distribution generated from the Weibull distribution function, the average inertial impaction efficiency for the distribution, and the efficiency for the mass median diameter of the distribution. Requires the Weibull parameter values of the distribution, particle density, collector diameter, fluid density and viscosity, and free-stream wind velocity.

Format for Input Cards

Card	Column	Format	Symbol	Denotation	Unit
1	1	I1	IOP	Option	
	2-80	79A1	ID	Data identification	
2	1-10	E10.0	RHO	Particle density	gm/cc
	11-20	E10.0	RHOA	Fluid density	gm/cc
	21-30	E10.0	RMU	Fluid viscosity	poise
3	1-10	F10.0	U	Free-stream velocity	cm/sec
	11-20	F10.0	D	Collector diameter	cm
4	1-10	F10.0	ETA	Weibull scale parameter	microns
	11-20	F10.0	BETA	Weibull shape parameter	
	21-30	F10.0	GAMMA	Weibull location parameter	microns

Sample Input



TEST CASE FOR OPTION FOUR
PARTICLE DENSITY = 1.0430+00 GM/CC   FLUID DENSITY = 1.2047-03 GM/CC
FLUID VISCOSITY = 1.8100-04 POISE   COLLECTOR DIAMETER = .9000 CM
FREE-STREAM VELOCITY = 357.600 CM/SEC
VELOCITY FIELD SCALING PARAMETER, PHI = 22.2678
WEIBULL PARAMETERS: SCALE = 34.4356 SHAPE = 1.7461 LOCATION = -1.7375
AVERAGE INERTIAL IMPACTION EFFICIENCY = .4666792

CUMULATIVE PER CENT MASS	MAXIMUM PART DIAM (MICRONS)	INTERVAL AVERAGE (MICRONS)	INERTIAL PARAMETER	INERTIAL IMPACTION EFFICIENCY
2	2.134			
		2.647	.0178	.0000000
3	3.160			
		3.607	.0331	.0000000
4	4.055			
		4.459	.0506	.0000000
5	4.864			
		5.238	.0698	.0000000
6	5.612			
		5.964	.0905	.0000000
7	6.315			
		6.648	.1124	.0000000
8	6.982			
		7.300	.1356	.0000111
9	7.619			
		7.925	.1598	.0015139
10	8.232			
		8.528	.1850	.0071181
11	8.824			
		9.112	.2112	.0161853
12	9.399			
		9.679	.2383	.0288266
13	9.959			
		10.232	.2664	.0429852
14	10.506			
		10.773	.2953	.0584635
15	11.041			
		11.303	.3250	.0748710
16	11.565			
		11.823	.3556	.0916102
17	12.081			
		12.335	.3871	.1084189
18	12.589			
		12.840	.4194	.1253332
19	13.090			
		13.337	.4525	.1422780
20	13.585			
		13.829	.4865	.1585318
21	14.073			
		14.315	.5213	.1749852
22	14.557			
		14.797	.5570	.1913652
23	15.037			
		15.274	.5935	.2068813

CUMULATIVE PER CENT MASS	MAXIMUM PART DIAM (MICRONS)	INTERVAL AVERAGE (MICRONS)	INERTIAL PARAMETER	INERTIAL IMPACTION EFFICIENCY
24	15.512			
		15.748	.6303	.2219787
25	15.984			
		16.219	.6592	.2369364
26	16.453			
		16.686	.7053	.2518321
27	16.919			
		17.151	.7484	.2665775
28	17.383			
		17.614	.7893	.2809987
29	17.845			
		18.076	.8312	.2948895
30	18.306			
		18.535	.8740	.3081317
31	18.765			
		18.994	.9178	.3207812
32	19.223			
		19.452	.9626	.3330823
33	19.681			
		19.909	1.0084	.345[illegible]
34	20.138			
		20.366	1.0552	.3581455
35	20.594			
		20.823	1.1037	.3703911
36	21.051			
		21.280	1.1527	.3823278
37	21.506			
		21.737	1.2029	.3939511
38	21.966			
		22.195	1.2533	.4052973
39	22.424			
		22.654	1.3058	.4163736
40	22.884			
		23.114	1.3592	.4271942
41	23.345			
		23.576	1.4141	.4377710
42	23.807			
		24.039	1.4701	.4481161
43	24.271			
		24.504	1.5275	.4582405
44	24.737			
		24.971	1.5867	.4681546
45	25.205			
		25.440	1.6465	.4778682
46	25.675			
		25.912	1.7081	.4873904
47	26.148			
		26.387	1.7713	.4967296
48	26.625			
		26.864	1.8360	.5058945
49	27.104			
		27.345	1.9023	.5148924

CUMULATIVE PER CENT MASS	MAXIMUM PART DIAM (MICRONS)	INTERVAL AVERAGE (MICRONS)	INERTIAL PARAMETER	INERTIAL IMPACTION EFFICIENCY
50	27.587		1.9361	.5193225 MMD
		27.830	1.9703	.5237306
51	28.073			
		28.318	2.0401	.5324161
52	28.564			
		28.811	2.1117	.5409554
53	29.058			
		29.308	2.1852	.5493548
54	29.558			
		29.810	2.2607	.5576204
55	30.062			
		30.317	2.3382	.5657579
56	30.572			
		30.829	2.4173	.5737727
57	31.087			
		31.347	2.4998	.5816702
58	31.608			
		31.871	2.5842	.5894555
59	32.135			
		32.402	2.6703	.5971336
60	32.669			
		32.940	2.7603	.6047093
61	33.210			
		33.485	2.8524	.6121873
62	33.759			
		34.038	2.9474	.6195723
63	34.316			
		34.599	3.0454	.6268686
64	34.882			
		35.170	3.1467	.6340808
65	35.457			
		35.749	3.2513	.6412132
66	36.042			
		36.339	3.3595	.6482703
67	36.637			
		36.940	3.4715	.6552563
68	37.243			
		37.553	3.5876	.6621757
69	37.862			
		38.177	3.7073	.6690328
70	38.493			
		38.815	3.8329	.6758322
71	39.138			
		39.468	3.9628	.6825782
72	39.798			
		40.135	4.0980	.6892757
73	40.473			
		40.819	4.2383	.6959295
74	41.166			
		41.521	4.3859	.7025444
75	41.877			
		42.242	4.5395	.7091258

CUMULATIVE PER CENT MASS	MAXIMUM PART DIAM (MICRONS)	INTERVAL AVERAGE (MICRONS)	INERTIAL PARAMETER	INERTIAL IMPACTION EFFICIENCY
76	42.608			
		42.984	4.700[illegible]	.7156791
77	43.360			
		43.748	4.[illegible]	.722210[illegible]
78	44.136			
		44.537	5.0462	.7287[illegible]54
79	44.938			
		45.353	5.[illegible]329	.7352315
80	45.769			
		46.199	5.4[illegible]98	.74173[illegible]7
81	46.630			
		47.078	5.[illegible]380	.7462463
82	47.525			
		47.992	5.[illegible]595	.7547723
83	48.459			
		48.946	6.0351	.7613240
84	49.430			
		49.948	6.3465	.7670128
85	50.460			
		51.000	6.6165	.77459[illegible]2
86	51.539			
		52.113	6.908[illegible]	.781[illegible]577
87	52.660			
		53.286	7.22[illegible]	.788[illegible]476
88	53.893			
		54.541	7.5[illegible]77	.79494[illegible]
89	55.189			
		55.886	7.9457	.801976[illegible]
90	56.584			
		57.341	8.[illegible]646	.8[illegible]1771
91	58.098			
		58.927	8.[illegible]303	.8165[illegible]11
92	59.757			
		60.677	9.3564	.8242775
93	61.598			
		62.637	9.[illegible]511	.8323191
94	63.675			
		64.873	10.7065	.8408314
95	66.071			
		67.435	11.6893	.850[illegible]10
96	68.919			
		70.594	12.7140	.860179[illegible]
97	72.463			
		74.863	14.2399	.87185[illegible]
98	77.268			

[illegible] PRINTS

CLAUDE.PELLEGRINO.IMPEF

```
      C     MAIN PROGRAM IMPEF
            DIMENSION PD(98),APD(97),RKS(97),E(97),ID(79)
            IPRT=0
      1     READ (5,34,END=29) IOP,ID
            IF (IOP.LT.1.OR.IOP.GT.4) GO TO 26
            GO TO (2,4,5,11), IOP
      2     READ (5,35) U,D,PDM
            RHO=1.0
            RHOA=1.2047E-3
            RMU=1.81E-4
      3     RK=(PDM*PDM*1.0E-8)*((RHO*U)/(9.0*RMU*D))
            PHI=(9.0*RHOA*RHOA*U*D)/(RMU*RHO)
            GO TO 6
      4     READ (5,36) RHO,RHOA,RMU,U,D,PDM
            GO TO 3
      5     READ (5,38) PHI,RK
      6     IF (IPRT) 7,7,8
      7     WRITE (6,32) ID
            GO TO 9
      8     WRITE (6,33) ID
      9     IF (IOP.EQ.3) GO TO 10
            WRITE (6,41) RHO,RHOA,RMU,D,U
            WRITE (6,42) PDM
      10    IF (RK.GT.0.13.AND.RK.LE.300.) GO TO 12
            IF (RK.LE.0.0.OR.RK.GT.300.) GO TO 27
            EFF=0.0
            GO TO 22
      11    READ (5,37) RHO,RHOA,RMU,U,D
            PHI=(9.0*RHOA*RHOA*U*D)/(RMU*RHO)
      12    IF (PHI.LT.0.0.OR.PHI.GT.10000.0) GO TO 28
            IF (PHI.LE.1.0) GO TO 13
            CALL NUMRTR (PHI)
      13    IF (IOP.NE.4) GO TO 21
            WRITE (6,32) ID
            WRITE (6,41) RHO,RHOA,RMU,D,U
            WRITE (6,43) PHI
            CHLD=((RHO*U)/(9.*RMU*D))*1.0E-8
            CALL WEIBUL (CHLD,RKMMD,PD,APD,RKS)
            SUME=0.0
            DO 16 I=2,97
            IF (RKS(I).LE.0.13) GO TO 14
            CALL LAGRNG (PHI,RKS(I),E(I))
            GO TO 15
      14    E(I)=0.0
      15    SUME=SUME+E(I)
      16    CONTINUE
            AVGE=SUME/96.
            WRITE (6,44) AVGE
            WRITE (6,31)
            WRITE (6,45)
            LINES=12
            DO 20 I=2,97
            LINES=LINES+2
            IF (I.EQ.50) GO TO 17
            WRITE (6,46) I,PD(I),APD(I),RKS(I),E(I)
            IF (LINES.LT.55) GO TO 20
```

```
      LINES=4
      WRITE (6,30)
      WRITE (6,45)
      GO TO 20
   17 IF (RKMMD.LE.0.13) GO TO 19
      CALL LAGRNG (PHI,RKMMD,EFF)
   18 WRITE (6,47) I,PD(I),RKMMD,EFF,APD(I),RKS(I),E(I)
      GO TO 20
   19 EFF=0.0
      GO TO 18
   20 CONTINUE
      I=98
      WRITE (6,46) I,PD(I)
      IPRT=-1
      GO TO 23
   21 CALL LAGRNG (PHI,RK,EFF)
   22 WRITE (6,39) PHI,RK,EFF
      IPRT=1
   23 IF (PHI-10.) 24,25,25
   24 WRITE (6,40)
   25 WRITE (6,31)
      GO TO 1
   26 WRITE (6,48) IOP
      GO TO 29
   27 WRITE (6,49) RK
      GO TO 29
   28 WRITE (6,50) PHI
   29 CALL EXIT
   30 FORMAT (1H1)
   31 FORMAT (/)
   32 FORMAT (1H1,79A1)
   33 FORMAT (1H ,79A1)
   34 FORMAT (I1,79A1)
   35 FORMAT (3F10.0)
   36 FORMAT (3E10.0/3F10.0)
   37 FORMAT (3F10.0/2F10.0)
   38 FORMAT (2F10.0)
   39 FORMAT (40H VELOCITY FIELD SCALING PARAMETER, PHI =,F11.4/24H INER
     1TIAL PARAMETER, K =,F9.4/32H INERTIAL IMPACTION EFFICIENCY =,F9.7)
   40 FORMAT (35H POTENTIAL FLUID FLOW MAY NOT APPLY)
   41 FORMAT (19H PARTICLE DENSITY =,1PE10.4,6H GM/CC,4X,15HFLUID DENSIT
     1Y =,1PE10.4,6H GM/CC/18H FLUID VISCOSITY =,1PE10.4,6H POISE,4X,
     220HCOLLECTOR DIAMETER =,0PF7.4,3H CM/23H FREE STREAM VELOCITY =,
     30PF9.3,7H CM/SEC)
   42 FORMAT (20H PARTICLE DIAMETER =,F8.3,8H MICRONS)
   43 FORMAT (40H VELOCITY FIELD SCALING PARAMETER, PHI =,F11.4)
   44 FORMAT (40H AVERAGE INERTIAL IMPACTION EFFICIENCY =,F9.7)
   45 FORMAT (3X,10HCUMULATIVE,6X,7HMAXIMUM,6X,8HINTERVAL,17X,
     18HINERTIAL/4X,6HPER CENT,6X,9HPART DIAM,5X,7HAVERAGE,4X,
     28HINERTIAL,6X,9HIMPACTION/6X,4HMASS,8X,3H(MICRONS),4X,9H(MICRONS),
     33X,9HPARAMETER,4X,10HEFFICIENCY)
   46 FORMAT (6X,I3,5X,F8.3/31X,F8.3,3X,F9.4,5X,F9.7)
   47 FORMAT (6X,I3,9X,F8.3,16X,F9.4,5X,F9.7,4H MMD/31X,F8.3,3X,F9.4,5X,
     1F9.7)
   48 FORMAT (7H OPTION,I2,17H IS NOT AVAILABLE)
   49 FORMAT (24H INERTIAL PARAMETER, K =,E14.9,13H OUT OF RANGE)
   50 FORMAT (40H VELOCITY FIELD SCALING PARAMETER, PHI =,E14.9,13H OUT
     1OF RANGE)
      END
```

```
CLAUDE,PELLEGRINO,WEIBUL                                                                    WEIBUL01
              SUBROUTINE WEIBUL (CHLD,RKMMD,PD,APD,RKS)                                     WEIBUL02
              DIMENSION PD(98),APD(97),RKS(97)                                              WEIBUL03
              READ (5,2) ETA,BETA,GAMMA                                                     WEIBUL04
              WRITE (6,3) ETA,BETA,GAMMA                                                    WEIBUL05
              POWER=1.0/BETA                                                                WEIBUL06
              DIFF=ETA-GAMMA                                                                WEIBUL07
              DO 1 INC=2,98                                                                 WEIBUL08
              PD(INC)=GAMMA+(DIFF*((ALOG(1.0/(1.0-(FLOAT(INC)/100.))))**POWER))             WEIBUL09
              IF (INC.EQ.2) GO TO 1                                                         WEIBUL10
              I=INC-1                                                                       WEIBUL11
              APD(I)=(PD(INC)+PD(I))/2.                                                     WEIBUL12
              RKS(I)=APD(I)*APD(I)*CHLD                                                     WEIBUL13
    1         CONTINUE                                                                      WEIBUL14
              RKMMD=PD(50)*PD(50)*CHLD                                                      WEIBUL15
              RETURN                                                                        WEIBUL16
    2         FORMAT (3F10.0)                                                               WEIBUL17
    3         FORMAT (28H WEIBULL PARAMETERS: SCALE =,F10.4,9H  SHAPE =,F7.4,               WEIBUL18
             112H  LOCATION =,F9.4)                                                         WEIBUL19
              END
```

Appendix

CLAUDE.PELLEGRINO.LAGRNG

```
      SUBROUTINE LAGRNG (PHI,RK,EFF)                                    LAGRNG01
      DIMENSION EN(13),ED(13),DIF(13),X(13),ELD(7),ELN(7),DIFL(7),Y(2), LAGRNG02
     1RNT(3)                                                            LAGRNG03
      DATA X/0.0,.788457356,1.60943791,2.30258510,3.09104245,3.91202301,LAGRNG04
     14.60517019,5.29831737,6.21460807,6.90775526,7.70074779,8.51719320,LAGRNG05
     29.21034038/ED/22357020.1,-1812698.47,269447.688,-88311.7591,      LAGRNG06
     341628.3997,-22779.8643,18228.2921,-24986.1752,45785.7457,-        LAGRNG07
     489020.3004,330206.731,-1864839.07,17170287.1/ELD/10098.2753,-     LAGRNG08
     51523.10928,610.97745,-479.26176,643.540984,-1501.68156,8921.7671/,LAGRNG09
     6RNT/0.22,0.5,1.0/                                                 LAGRNG10
      KIN=1                                                             LAGRNG11
    1 IF (RK.LT.RNT(KIN)) GO TO 2                                       LAGRNG12
      KIN=KIN+1                                                         LAGRNG13
      IF (KIN-3) 1,1,2                                                  LAGRNG14
    2 RKLX=ALOG(RK)                                                     LAGRNG15
      IF (PHI.LE.1.0) GO TO 5                                           LAGRNG16
      EFF=0.0                                                           LAGRNG17
      INC=1                                                             LAGRNG18
      IF (RK.GT.64.0) INC=2                                             LAGRNG19
      DO 4 N=2,14,INC                                                   LAGRNG20
      CALL POLY (N,KIN,RKLX,E)                                          LAGRNG21
      IF (INC.EQ.2) GO TO 3                                             LAGRNG22
      EFF=EFF+EN(N-1)/ED(N-1)*E                                         LAGRNG23
      GO TO 4                                                           LAGRNG24
    3 I=N/2                                                             LAGRNG25
      EFF=EFF+ELN(I)/ELD(I)*E                                           LAGRNG26
    4 CONTINUE                                                          LAGRNG27
      RETURN                                                            LAGRNG28
    5 DO 6 N=1,2                                                        LAGRNG29
      CALL POLY (N,KIN,RKLX,E)                                          LAGRNG30
      Y(N)=E                                                            LAGRNG31
    6 CONTINUE                                                          LAGRNG32
      EFF=Y(1)*(1.-PHI)+Y(2)*PHI                                        LAGRNG33
      RETURN                                                            LAGRNG34
      ENTRY NUMRTR(PHI)                                                 LAGRNG35
      PHIL=ALOG(PHI)                                                    LAGRNG36
      DO 7 I=1,13                                                       LAGRNG37
      DIF(I)=(PHIL-X(I))                                                LAGRNG38
      IF (I.GT.7) GO TO 7                                               LAGRNG39
      DIFL(I)=(PHIL-X(2*I-1))                                           LAGRNG40
    7 CONTINUE                                                          LAGRNG41
      DO 9 I=1,13                                                       LAGRNG42
      ENUM=1.0                                                          LAGRNG43
      ELNUM=1.0                                                         LAGRNG44
      DO 8 J=1,13                                                       LAGRNG45
      IF (I.EQ.J) GO TO 8                                               LAGRNG46
      ENUM=DIF(J)*ENUM                                                  LAGRNG47
      IF (I.GT.7.OR.J.GT.7) GO TO 8                                     LAGRNG48
      ELNUM=DIFL(J)*ELNUM                                               LAGRNG49
    8 CONTINUE                                                          LAGRNG50
      EN(I)=ENUM                                                        LAGRNG51
      IF (I.GT.7) GO TO 9                                               LAGRNG52
      ELN(I)=ELNUM                                                      LAGRNG53
    9 CONTINUE                                                          LAGRNG54
      RETURN                                                            LAGRNG55
      END                                                               LAGRNG56
```

CLAUDE.PELLEGRINO.POLY

```
      SUBROUTINE POLY (N,KIN,X,E)                                       POLY0001
      DIMENSION NCOF(56),P(14,4,8)                                      POLY0002
      DATA NCOF/6,7,7,8,6,6,6,7,6,7,7,7,6,7,7,8,6,6,7,7,6,7,7,7,6,6,7,7,POLY0003
     16,6,6,8,6,7,7,8,6,6,6,7,5,8,7,6,6,7,7,8,6,7,7,8,5,8,7,8/           POLY0004
C     PHI = 0                                                           POLY0005
      DATA (P(1,1,I),I=1,6)/-12.200402,-35.459569,-297.18542,-462.32274,POLY0006
     1-360.06598,-111.55255/(P(1,2,J),J=1,7)/62.348075,395.99417,       POLY0007
     21030.9451,1408.1269,1064.5792,421.75102,68.7841330/(P(1,3,K),K=1, POLY0008
     37)/-62.940926,-120.18487,-85.164326,-27.596719,-3.9510734,-       POLY0009
     4.97751282,-3.9181877E-2/(P(1,4,L),L=1,8)/-1.0536954E-5,1.9332078E-POLY0010
     54,-1.6152915E-3,8.0885181E-3,-2.2809549E-2,4.0923174E-3,-         POLY0011
     6.79783478,-3.9181353E-2/                                          POLY0012
C     PHI = 1                                                           POLY0013
      DATA (P(2,1,I),I=1,6)/4.9845675,52.832639,210.59407,403.96104,    POLY0014
     1376.12181,137.7212/(P(2,2,J),J=1,6)/-3.6211216E-2,-.17775906,-    POLY0015
     2.54651109,-.75115414,-1.2866331,-.13762106/(P(2,3,K),K=1,6)/-     POLY0016
     31.7649616,-.28058164E1,-1.4690678,-.22811208,-.77448736,-         POLY0017
     41.1119515E-2/(P(2,4,L),L=1,7)/9.3879216E-5,-1.4651483E-3,         POLY0018
     59.6292892E-3,-2.9244455E-2,1.8175188E-2,-.78756638,-1.1119545E-2/ POLY0019
C     PHI = 2.2                                                         POLY0020
      DATA (P(3,1,I),I=1,6)/-26.205198,-212.34232,-668.74366,-1116.7436,POLY0021
     1-905.75446,-293.15808/(P(3,2,J),J=1,7)/-2.9697585,-19.546255,-    POLY0022
     252.775689,-74.952536,-58.950642,-25.135597,-4.1293537/(P(3,3,K),K=POLY0023
     31,7)/1.6140825,1.0850372,-.96742653,-.93003128,-.15264606,-       POLY0024
     4.76202787,2.4042651E-4/(P(3,4,L),L=1,7)/2.0318435E-5,-5.9103702E- POLY0025
     54,5.5930552E-3,-2.1070663E-2,1.2375939E-2,-.77654255,2.4063257E-4/POLY0026
C     PHI = 5                                                           POLY0027
      DATA (P(4,1,I),I=1,6)/-26.212424,-214.28974,-701.61294,-1148.349,-POLY0028
     1941.31809,-307.88626/(P(4,2,J),J=1,7)/.45517389,2.6995224,        POLY0029
     26.5915579,8.3665223,5.9148874,1.4364731,.3573586/(P(4,3,K),K=1,7)/POLY0030
     3-5.8102354,-14.826274,-13.629123,-5.4803251,-.8580772,-.78651089, POLY0031
     41.7312087E-2/(P(4,4,L),L=1,8)/-3.6119607E-5,6.3913339E-4,-        POLY0032
     54.7997086E-3,2.0367387E-2,-4.7588177E-2,3.7991413E-2,-.77708922,  POLY0033
     6.01731202/                                                        POLY0034
C     PHI = 10                                                          POLY0035
      DATA (P(5,1,I),I=1,6)/-50.664987,-423.64833,-1415.6474,-2361.2007,POLY0036
     1-1965.8164,-652.49849/(P(5,2,J),J=1,6)/-.062968392,-.19555374,-   POLY0037
     2.24037953,.011244131,-.60911288,.097500309/(P(5,3,K),K=1,7)/      POLY0038
     31.4597121E-2,9.49348899E-2,.28102331,.3238649,.20596467,-         POLY0039
     4.70590523,3.7057228E-2/(P(5,4,L),L=1,7)/9.6683056E-5,-1.6339242E- POLY0040
     53,1.1191886E-2,-3.5935414E-2,3.336479E-2,-.7662405,3.7057191E-2/  POLY0041
C     PHI = 22                                                          POLY0042
      DATA (P(6,1,I),I=1,6)/-76.950465,-651.69666,-2206.645,-3732.3263,-POLY0043
     13153.5891,-1063.8574/(P(6,2,J),J=1,7)/5.7644622,37.522245,        POLY0044
     2100.76219,142.69011,112.43496,45.959403,8.0517842/(P(6,3,K),K=1,7)POLY0045
     3/-4.7767558,-12.444464,-11.500556,-4.5950604,-.68987081,-         POLY0046
     4.74831659,6.6480331E-2/(P(6,4,L),L=1,7)/1.7177586E-4,-2.6882966E- POLY0047
     53,1.6780465E-2,-4.9847116E-2,4.9792698E-2,-.75707841,6.648065E-2/ POLY0048
C     PHI = 50                                                          POLY0049
      DATA (P(7,1,I),I=1,6)/-59.938282,-501.29199,-1675.1358,-2793.8696,POLY0050
     1-2325.6901,-771.90025/(P(7,2,J),J=1,6)/-.40153928,-1.7185120,-    POLY0051
     22.6991333,-1.6266652,-.89185129,.21709087/(P(7,3,K),K=1,7)/       POLY0052
     35.7990554E-2,.31663007,.67074199,.61650862,.30036899,-.66225648,  POLY0053
     4.10659907/(P(7,4,L),L=1,7)/1.853719E-4,-2.7976853E-3,1.7395729E-2,POLY0054
     5-5.069296E-2,5.2031505E-2,-.73985798,.10659903/                   POLY0055
C     PHI = 100                                                         POLY0056
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      DATA (P(8,1,I),I=1,6)/-17.064731,-135.07757,-427.92071,-677.22163,POLY0057
     1-535.96207,-168.81051/(P(8,2,J),J=1,6)/.34226461,2.4416096,       POLY0058
     26.3623809,7.9466487,4.0134799,1.2230111/(P(8,3,K),K=1,6)/-        POLY0059
     36.3096724,-9.4214242,-4.5663823,-.73041192,-.71550948,.14857452/  POLY0060
     4(P(8,4,L),L=1,8)/9.0060305E-5,-1.2733944E-3,6.5331556E-3,-        POLY0061
     51.2515114E-2,-3.0812454E-3,1.7035729E-2,-.71368247,.14657444/     POLY0062
C     PHI = 200                                                         POLY0063
      DATA (P(9,1,I),I=1,6)/-42.396161,-355.31647,-1192.8835,-2003.9678,POLY0064
     1-1684.9216,-566.20499/(P(9,2,J),J=1,7)/1.0660894,8.7591944,       POLY0065
     228.501044,47.240840,42.393325,18.867897,3.8321726/(P(9,3,K),K=1,7)POLY0066
     3/-39.460761,-80.456781,-60.748291,-20.641773,-2.9597241,-         POLY0067
     4.80941524,.19835457/(P(9,4,L),L=1,8)/-3.4881657E-4,5.4965192E-3,- POLY0068
     53.465441E-2,.11210716,-.19695587,.16083490,-.7357441,.19805452/   POLY0069
C     PHI = 500                                                         POLY0070
      DATA (P(10,1,I),I=1,6)/-12.020613,-92.568931,-284.75887,-         POLY0071
     1436.40904,-333.62156,-100.80133/(P(10,2,J),J=1,6)/-1.4257366,-    POLY0072
     28.0514717,-18.340894,-19.614001,-11.357255,-1.9831040/(P(10,3,K), POLY0073
     3K=1,6)/4.2113941,7.1224666,4.1790245,1.0121884,-.58806253,        POLY0074
     4.27458546/(P(10,4,L),L=1,7)/1.9737683E-4,-2.7265578E-3,1.4653205E-POLY0075
     52,-3.7207501E-2,2.6369855E-2,-.66753359,.27450544/                POLY0076
C     PHI = 1000                                                        POLY0077
      DATA (P(11,1,I),I=1,5)/14.829388,97.320297,240.33938,263.37799,   POLY0078
     1109.15044/(P(11,2,J),J=1,8)/17.029397,137.42034,470.41882,        POLY0079
     2895.10322,987.87372,653.645,236.55692,36.739044/(P(11,3,K),K=1,7)/POLY0080
     347.983167,101.37274,80.5671,29.548404,4.8616024,-.38993037,       POLY0081
     4.34078307/(P(11,4,L),L=1,8)/-5.0706425E-5,4.594885E-4,-1.3181914E-POLY0082
     53,1.7151807E-3,-2.9558258E-3,-1.233578E-2,-.63346949,.340783/     POLY0083
C     PHI = 2210                                                        POLY0084
      DATA (P(12,1,I),I=1,6)/-63.364116,-536.67989,-1818.2796,-         POLY0085
     13078.5398,-2604.6811,-879.71803/(P(12,2,J),J=1,7)/6.4439511,      POLY0086
     241.998908,112.46264,156.04802,122.55226,49.373179,8.7539036/(P(12,POLY0087
     33,K),K=1,7)/-36.484327,-77.532174,-62.802408,-24.102723,-         POLY0088
     44.3251476,-.9208608,.427815/(P(12,4,L),L=1,8)/-3.7367973E-6,      POLY0089
     53.6233045E-5,-2.0078772E-4,2.1297687E-3,-1.0367251E-2,2.6019194E- POLY0090
     63,-.62493408,.42781593/                                           POLY0091
C     PHI = 5000                                                        POLY0092
      DATA (P(13,1,I),I=1,6)/-103.53582,-863.60206,-3015.1879,-         POLY0093
     15139.6048,-4376.3562,-1487.8046/(P(13,2,J),J=1,7)/4.6788015,      POLY0094
     231.950727,89.903464,133.11754,109.29784,46.482059,8.8405497/(P(13,POLY0095
     33,K),K=1,7)/34.132168,69.52454,53.281683,19.049472,3.2204642,-    POLY0096
     4.38616475,.53065892/(P(13,4,L),L=1,8)/-4.4636683E-5,7.1149327E-4,-POLY0097
     54.7031388E-3,1.7822142E-2,-4.2369884E-2,4.4434352E-2,-.63295244,  POLY0098
     6.53065886/                                                        POLY0099
C     PHI = 10000                                                       POLY0100
      DATA (P(14,1,I),I=1,5)/10.993253,71.125908,173.31109,187.3833,    POLY0101
     177.147214/(P(14,2,J),J=1,8)/10.660898,84.798844,266.49179,        POLY0102
     2532.49198,587.36483,384.06109,137.03167,21.438196/(P(14,3,K),K=1, POLY0103
     37)/-16.237284,-32.2699,-24.002293,-8.3489724,-1.3820594,-         POLY0104
     4.69722816,.61335532/(P(14,4,L),L=1,8)/-1.0477396E-4,1.6059967E-3,-POLY0105
     59.7271265E-3,3.050984E-2,-5.4851738E-2,4.4704111E-2,-.60594781,   POLY0106
     6.61335528/                                                        POLY0107
      NS=4*(N-1)+KIN                                                    POLY0108
      M=NCOF(NS)                                                        POLY0109
      RLLRE=P(N,KIN,1)                                                  POLY0110
      DO 1 I=2,M                                                        POLY0111
      RLLRE=RLLRE*X+P(N,KIN,I)                                          POLY0112
    1 CONTINUE                                                          POLY0113
```

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      E=1./(EXP(EXP(RLLRE)))                                    POLY0114
      RETURN                                                    POLY0115
      END                                                       POLY0116
```

0BRKPT PRINTS

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